LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - PHYSICS

SECOND SEMESTER – APRIL 2013

PH 2815 - MATHEMATICAL PHYSICS - II

Date : 29/04/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART A

Answer **ALL** questions 10 x 2 = 20

1) Determine the Laplace transform of $t^2 e^{-3t}$ using the first shifting theorem.
2) Find the Fourier transform of the Dirac delta function.
3) Define finite Fourier cosine transform and give its inversion formula.
4) State the possible initial and boundary conditions of Laplace equation.
5) Write down the Laguerre's differential equation.
6) Define Fresnel integrals.
7) What is irreducible representation of a group?
8) State the great orthogonality theorem in the theory of group representation.
9) State the addition law of probability for mutually exclusive events. Illustrate with a simple example.
10) Write down the normal distribution function with the symbols explained

PART B

Answer any **FOUR** questions 4 x 7.5 = 30

11) (a) Find the Fourier transform of f(x) = $e^{-x^2/2}$. (b) Find the Fourier transform of f(x) = $e^{-a|x|}$.
12) Determine the Laplace transform of the function $\frac{\partial^2 u}{\partial x \partial t}$ given that L { u(x,t)} = U (x,s).
13) Show that (i) erf (-x) = - erf x and (ii) | erf x| ≤ 1
14) Work out the multiplication table of the symmetry group of the proper covering operations of an equilateral triangle. Write down all the subgroups and divide the group elements into classes. What are the allowed dimensionality of the representation matrices of the group?
15) A bag contains four white and two black balls and a second bag contains three of each colour. A bag is selected at random, and a ball is then drawn at random from the bag chosen. What is the probability that the ball drawn is white?

PART C

Answer any **FOUR** questions 4 x 12.5 = 50

16) Solve the system of two differential equations $\frac{d^2y_1}{dt^2}$ = $-ky_1 + k(y_2-y_1)$ and $\frac{d^2y_2}{dt^2}$ = $-k(y_2-y_1) - ky_2$ with the initial conditions $y_1(0) = 1$, $y_2(0) = 1$, $y_1'(o) = \sqrt{(3k)}$, $y_2'(0) = -\sqrt{(3k)}$.

17) Solve the IBVP described as PDE : $u_{xx} = \frac{1}{c^2} u_{tt} - \cos \omega t$, $0 \le x < \infty$, $0 \le t < \infty$; BCs: $u(0,t) = 0$, u is bounded as x tends to ∞ and ICs: $u_t(x,0) = u(x,0) = 0$.

18) Establish the orthogonality property of the Laguerre polynomial by the use of its generating function.

19) (a) Obtain the transformation matrices of the symmetry elements (i) for reflection in xy-plane and (ii) for the centre of symmetry . (b) Explain the following main features of the axial rotation group SO (2): (i) Law of composition, identity element and the inverse (ii) The representation of the group (iii) The character of the representation with its orthogonality theorem.

20) (a) Prove that the Poisson distribution is a particular limiting form of the binomial distribution.

(b) Determine the mean of the Poisson distribution.

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